A Simple Device and a Project for the Nonlinear Control Systems Laboratory

Jose Maria Giron-Sierra, *Member, IEEE*

***Abstract*—This paper describes a new inverted pendulum system that is useful to illustrate important aspects of nonlinear control systems theory. The stabilization of the pendulum is to be achieved by the on–off action of two electromagnets; therefore, an adequate switching policy has to be applied. The pendulum can be controlled by a computer, using simple electronic interface circuits. This paper considers first an example from literature that suggests some design principles about the new system. Second, there is a detailed description of the pendulum. Finally, the use of the system in a student's project is presented. The experimental device is easy to build, inexpensive, and has good pedagogical impact.**

***Index Terms*—Automatic control teaching, computer control, experimental systems, nonlinear control.**

I. Introduction

NONLINEARITIES are important aspects to be considered in the teaching of control systems. There are at least two reasons for this fact. First, the practical nature of the processes to be controlled includes nonlinear phenomena. Second, the optimal steering of a process may imply nonlinear strategies. In addition, nonlinear systems attract considerable interest from the perspective of the study of complex and chaotic behaviors.

An essential part of our educational activity takes place in the laboratory. It is our desire to have experimental devices with a high didactic and motivational impact, simple to use and, if possible, inexpensive. The objective of this paper is to introduce an experimental device, easy to build and well suited for nonlinear control study in the laboratory. The device offers a good practical case for projects.

Our laboratory brings didactic support for computer engineering students who choose a course on digital control. Considering the characteristics of such students, the course concentrates on the following aspects:

- computer interaction with external systems: sensors, signals, sampling, D/A and A/D;
- real-time issues;
- operator and process interface software;
- automatic control principles and strategies.

Most students tend to think in computer (hard/soft) terms only. A recent questionnaire has shown that students feel the need for more knowledge about real processes and systems. This need is made more evident when starting to work on control problems. The author takes it into account, and tries to provide some realistic challenges in the laboratory, in order to promote the necessary insight.

Manuscript received November 30, 1998; revised May 17, 2000.

The author is with the Department Arquitectura de Computadores y Automatica, Fac. Ciencias Fisicas, Universidad Complutense de Madrid, 28040 Madrid, Spain (e-mail: gironsi@dia.ucm.es).

Publisher Item Identifier S 0018-9359(01)03866-3.

This paper is organized as follows. Section II considers an illustrative example from literature. The idea of the experimental device is inspired in this example. Section III describes the device—an inverted pendulum with ON–OFF actuators. A project was proposed to some students, based on this device. To be able to carry out the project, the students needed the pertinent background. Section IV presents the project and the educational methodology devised to get the required background. Section V comments on the results of the project.

II. An Example from the Literature

Many books on optimal control, when dealing with minimum-time problems, provide as an example a double integrator system [1]–[3]. An interesting version of the example is presented in [4]. It considers a system obeying Newton's laws as illustrated by the case of the one-dimensional attitude control of a satellite in its pitch plane. By means of a pair of reaction jets, the satellite has to point an antenna toward certain stellar objects [Fig. 1(a)]. Each jet is of ON–OFF type.

The case is mathematically stated with a simple equation

$$I \cdot \ddot{\theta} = \tau \tag{1}$$

where

$$\theta = \text{angle}; \tau = \text{torque}; I = \text{moment of Inertia}.$$

There are several control strategies that can be applied. The discussion of alternatives is made using the phase plane (the system is of second order). The state variables are chosen in the normal form

$$x_1 = \theta\,;\, x_2 = \dot{x}_1\,;\, u = \frac{\tau}{I}. \tag{2}$$

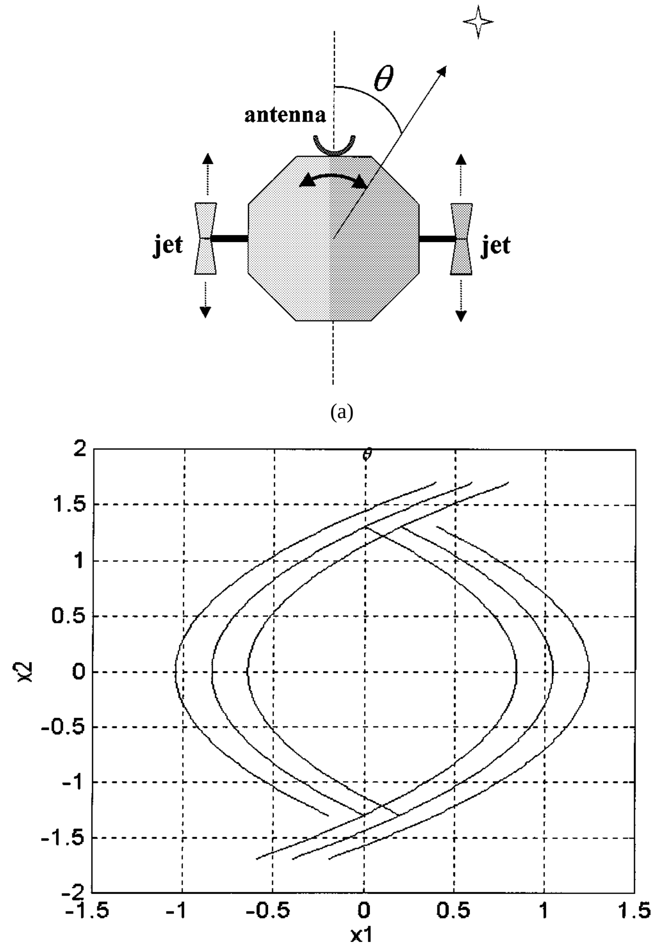
The system input can only take one of three alternative values

$$u = \begin{cases} +U \\ 0 \\ -U. \end{cases} \tag{3}$$

With these variables, the equations describing the dynamics of the system are

$$\dot{x}_1 = x_2\,;\, \dot{x}_2 = u. \tag{4}$$

The axes of the phase plane are angle (abscissa) and angular speed (ordinate). When jets fire, the state of the system changes, following a trajectory. The plot of this trajectory on the phase plane is a curve (a parabola) starting from the initial condition. The objective of the control is to drive the system, using the jets, from any initial condition, to the origin. There are two families of curves, one associated with $u = +U$, and the other to $u =$



(a)

(b)

Fig. 1. (a) Diagram of the satellite example. (b) Two families of responses on the phase-plane.

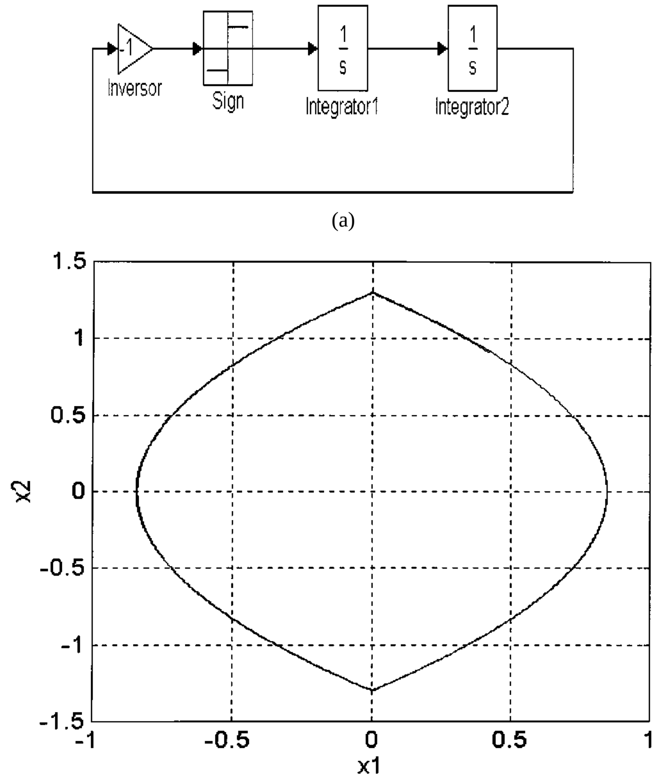
$-U$. Fig. 1(b) shows some members of the two families. Only one member of each family crosses the origin.

Dealing with this example, a sequence of attempts can be followed, going deeply into the problem. A trial-and-error start can be a first, spontaneous, strategy. It involves the firing of the jets as soon as a deviation of the antenna with respect to the object is detected. This procedure means to employ the vertical axis of the phase plane as a switching line. When the satellite deflects to the left or to the right, the jets are fired trying to counteract this movement. To accomplish this idea, a feedback loop is established, including a nonlinear block. Fig. 2(a) uses SIMULINK to depict the block diagram.

When this scheme is applied, starting from any initial condition apart from the origin, the results are as shown by Fig. 2(b). The failure is evident. Instead of stabilizing the satellite, a sustained oscillation is obtained. The junction of two parabolas forms a limit cycle.

Some predictive capability has to be assumed by the control, to counteract in anticipation the effect of inertia. A simple idea is to rotate the switching line. This is done by means of an ideal lead compensation, inserted into the feedback loop [Fig. 3(a)].

As shown by Fig. 3(b) the consequence is that from whichever initial condition, the control succeeds in driving the system to



(a)

(b)

Fig. 2. (a) SIMULINK diagram for an initial control strategy. (b) Phase-plane with the result: sustained oscillations.

the origin. The mechanism is to switch from one to another parabola, until arriving at the origin.

The use of ON–OFF type actuators is one of the reasons cited explicitly by Utkin [5], to justify the "sliding-mode" control. This type of actuator is commonly used in industry and other application fields. Slotine's book [6] on nonlinear control devotes an important part to the "sliding-mode" control.

The example in [4] continues by considering the minimum-time strategy called "bang-bang," which constitutes an interesting step in the scientific history of the development of optimal control theory.

III. Inverted Pendulum with ON-OFF Actuators

Considering the virtues of the example just described, the author resolved to build an experimental device with similar behavior. The inverted pendulum is a famous case [7]–[9] because of its inherent unstability. It is also of interest when studying complex or chaotic dynamics (especially when there are forcing terms). There is a great amount of literature concerning pendulum-based systems in general, and the inverted pendulum in particular. In their introduction Chen and Chen [10] remark that the control of the inverted pendulum has many points in common with the control of a rocket and also with the control of single-link manipulators in robotics [11]. One of the most popular versions of the inverted pendulum is the cart-pole setup [7], [12], [13], more difficult to control than this experimental device. This setup is frequently employed by the

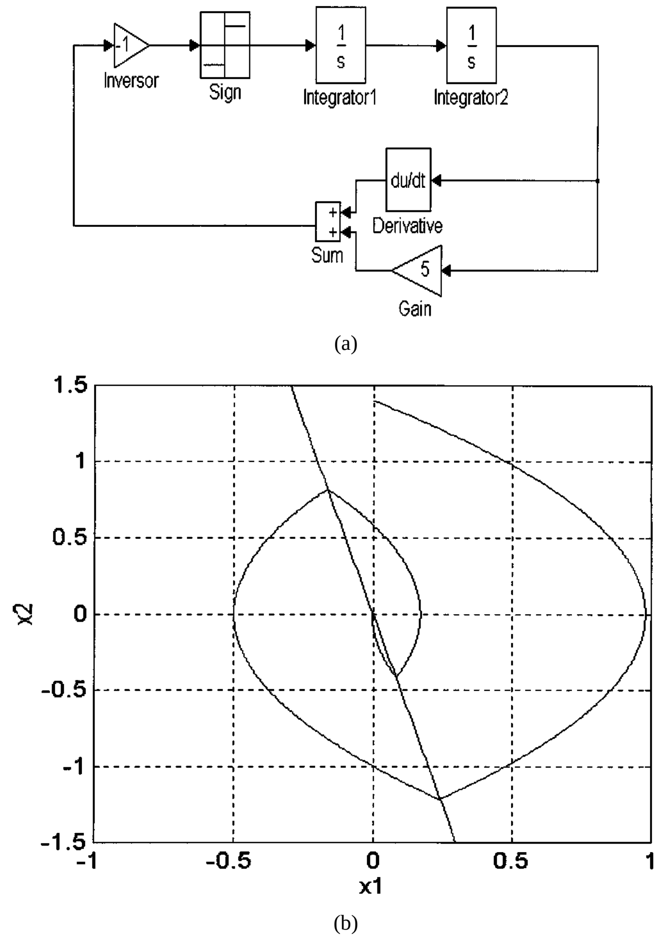


Fig. 3. (a) SIMULINK diagram for a better control strategy, with a rotated switching line. (b) Phase-plane with the result: the target is attained.

research on artificial intelligence techniques for control (fuzzy logic, neural networks, etc.) [14]–[16].

The distinctive feature of this device is that the pendulum has to be kept vertical, *by using two electromagnets*, operating with an on–off regime. The fulcrum is fixed (i.e., not on a cart, etc.). The rod is made of iron. The electromagnets are also fixed, near the rod, one to the left, the other to the right. Each electromagnet attracts the rod to its side, as soon as dc current is applied to the electromagnet.

This device has dynamic characteristics similar to the satellite (it also obeys Newton's laws), but is subject to different forces—the combined action of gravity and the electromagnets. The problem to be solved by the students is to design a control strategy to maintain the rod's vertical position, using the electromagnets. These on–off actuators are under computer control. To verify a strategy, a real-time control program must be developed, and tested in the laboratory. The pendulum can serve as a testbed for the experimental study of several control alternatives.

The setup consists of a mechanical system, interface electronic circuits, and a computer with an A/D and D/A board.

As illustrated in Fig. 4, the pendulum consists of a $L = 70$ cm.-long rod that rotates in the vertical plane. The rod is made with a hollow lightweight iron bar, 1 c. diameter, and weighing 200 gr (m). The two electromagnets are symmetrically placed with respect to the vertical position of the rod, one to the left, and the other one to the right. Each of them at $d = 5$ mm. from the rod surface when the rod is in the vertical position. The electromagnets are $l = 6$ cm over the fulcrum. The movement of the rod is restricted by the electromagnets themselves, as bumpers. Since the electromagnetic forces involved are of short-range effect, the electromagnets must also be near the rod. To get wide oscillations, the electromagnets must also be near the fulcrum.

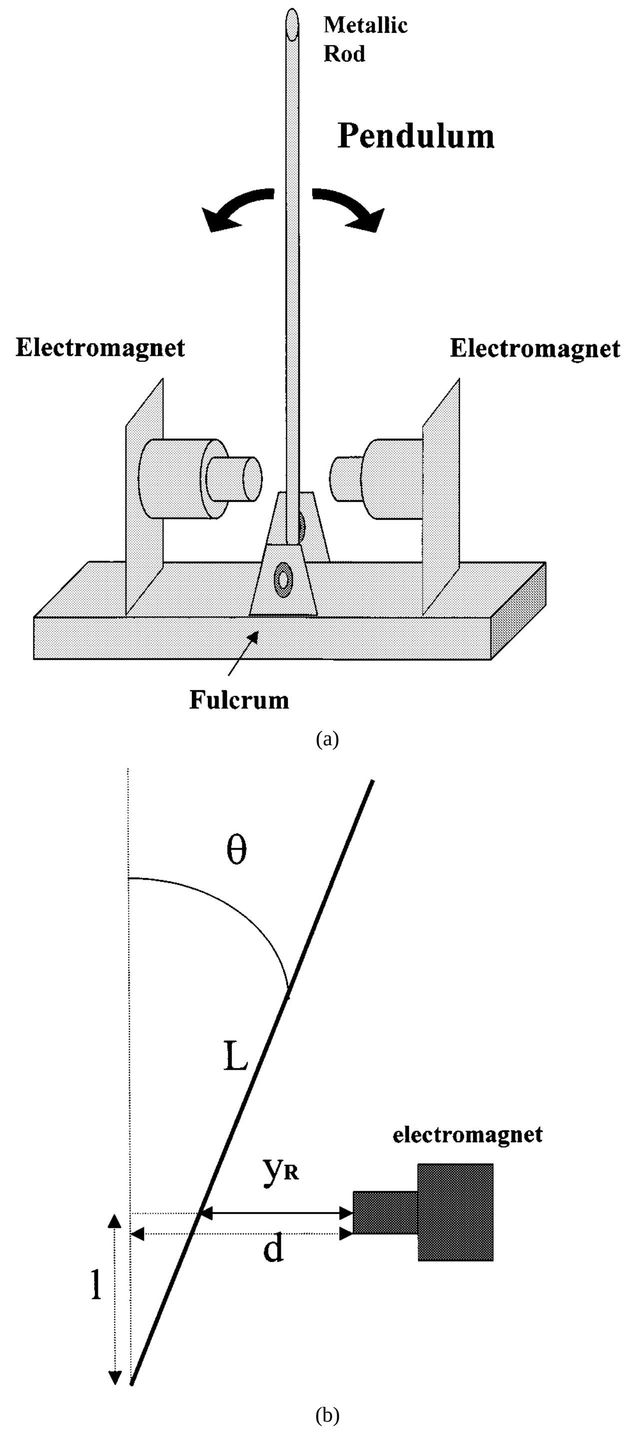


Fig. 4. (a) The inverted pendulum, with electromagnetic actuators. (b) Variables for the equations of the pendulum.

The forces exerted by the electromagnets on the rod can be expressed as in [17] with reference to Fig. 4:

$$F_{MR} = K_M \cdot e^{-y_R/a}\,;\; y_R = d - l \cdot \tan\theta$$
$$F_{ML} = K_M \cdot e^{-y_L/a}\,;\; y_L = d + l \cdot \tan\theta \tag{5}$$

where:

$K_M = 12Nw.$; $a = 0.006\,m.$; (determined empirically).

The torques due to gravity (τ_G) and magnetic force (τ_M) are the following:

$$\tau_G = \tfrac{1}{2} \cdot m \cdot L \cdot g \cdot \sin x_1$$
$$\tau_M = \begin{cases} +F_{MR} \cdot l \\ 0 \\ -F_{ML} \cdot l. \end{cases} \tag{6}$$

When designing the system, it is important to ensure that

$$|\tau_M| > |\tau_G| \quad \text{for} \quad x_1 = x_{1\,\max} \tag{7}$$

(the motions are limited by the electromagnets, so there is a maximum angle).

The total torque and the moment of Inertia are

$$\tau = \tau_G + \tau_M\,;\; I = \tfrac{1}{3} \cdot m \cdot L^2. \tag{8}$$

The same equation (1) can be used to describe the ideal behavior of the pendulum (neglecting friction, etc.). Also, the state variables can be chosen as in (2). With state variables, the equations describing the dynamics of the system are the following:

$$\dot{x}_1 = x_2\,;\; \dot{x}_2 = u \quad \text{for} \quad x_1 \le x_{1\,\max}. \tag{9}$$

Best measurements of the movement can be obtained by observing the top of the pendulum, where longitudinal motion is larger. So a sensor was built based on a row of 40 small photo-resistors, put side by side, and a long tubular incandescent lamp placed parallel to the row of photo-resistors. A narrow passage is formed between this row and the lamp. With an opaque rectangle, fastened to the top of the rod and moving in the passage, more or less light coming to the photo-resistors can be intercepted. In this way, we can measure the rod's position. The lamp is connected to filtered dc current. The lamp and the photo-resistors were placed in an opaque box, with a slit for the rod. The box is fixed with an auxiliary metallic bar over the top of the pendulum. Fig. 5 shows a photograph of the box, taken from the base.

A computer equipped with a standard 12 bit A/D and D/A board which also includes digital I/O channels is used. Interface electronics must be added externally to interact with the pendulum. The students designed and built these circuits (Fig. 6). With off-the-shelf power it is easy to activate/deactivate the electromagnets, amplifying the digital output signals from the computer. The variations of resistance in the set of photo-resistors are converted to voltage variations, by means of an operational amplifier. A second operational amplifier is used to differentiate this voltage signal in order to get a measurement of velocity. Fig. 7 shows a photograph of the pendulum with the sensor and the interface electronics. The complete device, including electronic circuits, is easy to build and inexpensive (less than $200 U.S.). As a control laboratory apparatus it provides an evident and clear problem to be solved. Fig. 8 shows a photograph of the experimental setup. The computer measures the status (position, velocity) of the pendulum through two analog input channels, and activates the electromagnets with two digital output channels.

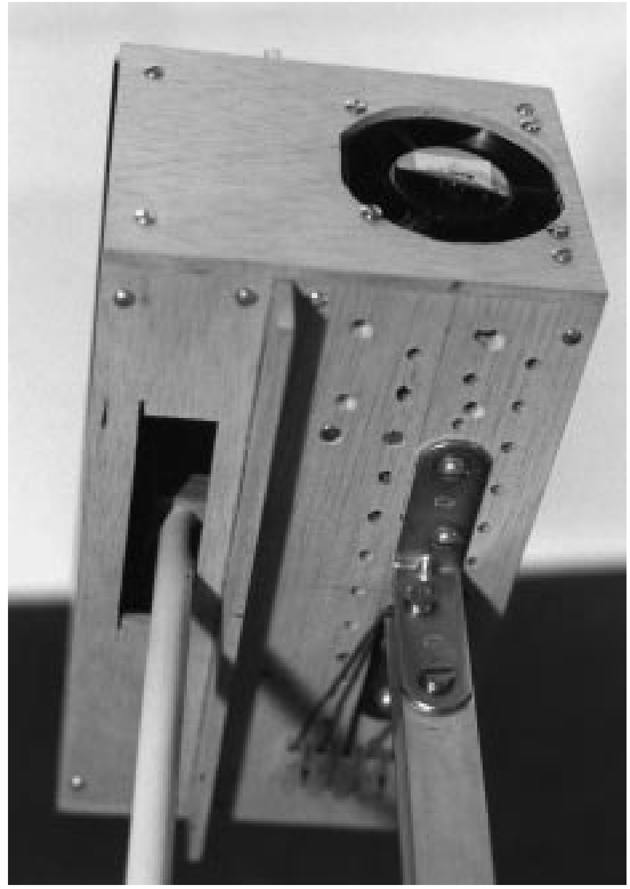


Fig. 5. The box containing the movement sensor: photograph from the bottom.

First tests of the system were made with two objectives: to calibrate the measurement system, and to develop the basic interface subroutines (that include the results of calibration). Then, a group of students, just before the lessons on nonlinear control, was asked to stabilize the pendulum. They started with the trivial approach described before, taking the ordinate axis as the switching line. The result is a limit cycle, with an impressive effect: the pendulum struck the bumpers violently and the system had to be stopped quickly. The students were disconcerted. Thinking the control failure was due to computation delays, they tried by various means to accelerate the response time of the computer. They got the same result. At that point, a theory session discussing the example was really welcomed by the students.

IV. Experimental Project for Students

According to a project-driven approach by our laboratory, an experimental project related to the inverted pendulum was proposed to our students. The objective was to develop a program

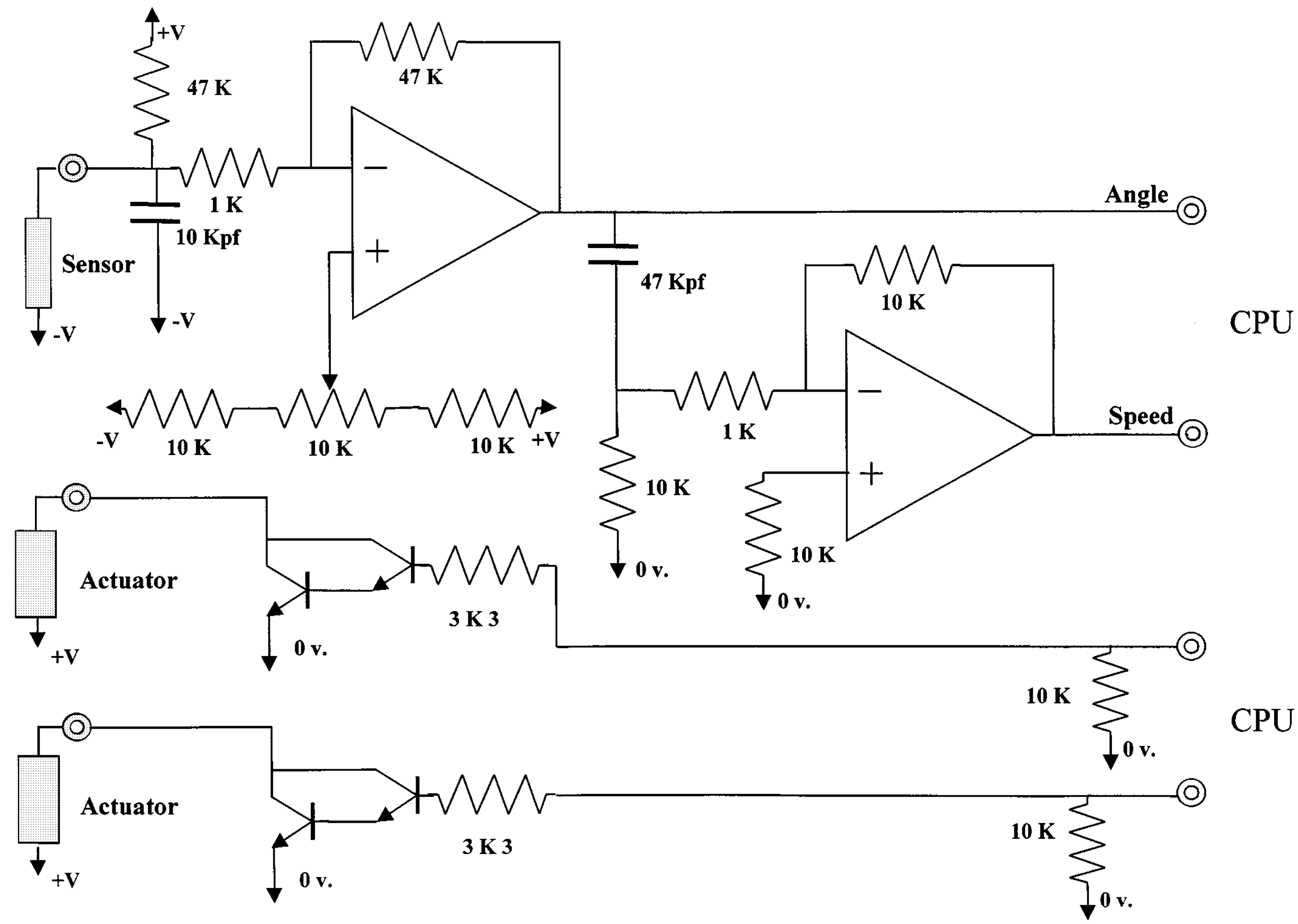


Fig. 6. Electronic circuits of the computer interface.

for the stabilization of the pendulum, using a switching line. The user of this program should have the opportunity to change the slope of the switching line, and to observe the experimental effects on the pendulum stabilization. In response to our project proposal, encouraging results were obtained. A group of students embarked on the project, worked with increasing motivation, and developed a good program.

The program development involved several fronts: to handle the A/D and D/A board, to dialogue with the user, to apply the digital control strategy, and to visualize the behavior of the pendulum. Complying with a structured methodology, it was not difficult to develop the program, especially when one has some experience with the necessary pieces. To acquire this experience the following laboratory exercises were done during the initial part of the course (before proposing the project):

— A signal generator was connected to an analog input of the A/D and D/A board. The students were asked to develop a program for the visualization, on the computer screen, of the signal from the generator.

— The objective of the second exercise was to develop a program that enabled the use of the computer as a signal generator, using the analog output of the A/D and D/A board. An oscilloscope was used to see the results.

— The third exercise was to develop a program for the computer to interact with switches and relays, through some given electronic interface circuits, using the digital inputs and outputs of the A/D and D/A board.

The students were required to use C++. They were provided with some simple program examples, in the form of documented listings, closely resembling to the objectives of the exercises.

Once they completed the exercises, the students saw the feasibility of the project. On the basis of the programing background acquired in the computer engineering courses, they went quite naturally to a modular building of the program. They used, as main parts, the code they developed for the exercises, with some ad-hoc adaptations. The students were given four laboratory sessions, 3 h each, to develop and test the program. Actually, since they also worked at home, the exact time they invested is not known. The final results were quite satisfactory, as will be described next.

V. Results of the Project

Before starting the project, the students were advised to keep things simple regarding both the use of the program and its code. The main reasons were to avoid excessive time dedicated to the project, and to obtain conveniently a simple, structured code as

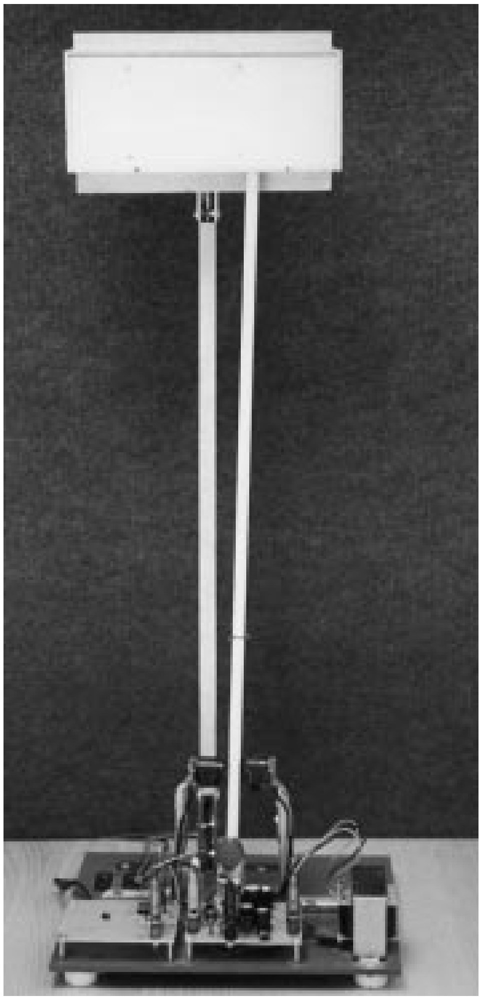


Fig. 7. Photograph of the inverted pendulum.

Fig. 8. Photograph of the complete setup.

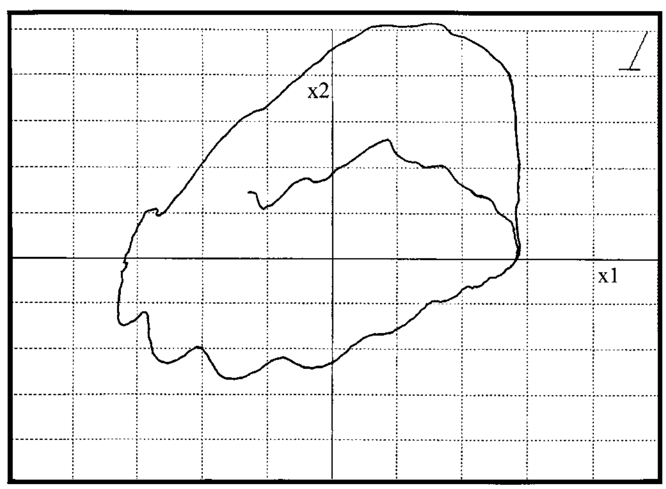


Fig. 9. A snapshot showing the good result of a rotated switching line.

a starting point for subsequent projects on new control strategies for the pendulum stabilization. So, plain MS-DOS was preferred to MS-Windows, with the display of points and curves provided by the Borland C++ graphics library.

After several steps of development and experimental testing, a simple and effective program was devised. Once this program is started, the user can specify the angle of the switching line. Then the program draws, on the screen, the axes of the phase-plane and begins to apply the control strategy to the pendulum. The program takes samples of the two signals coming from the sensor (position and velocity) at 1/20 s intervals. Each new sample is used to display in real-time a new point on the phase-plane. At any time, the program can be interrupted, letting the pendulum rest on one of the bumpers. Later on, it can be restarted. In general, the application of a rotated switching line drives the pendulum to an equilibrium state after few oscillations. The program displays the position of this switching line over the phase-plane. Fig. 9 shows an example of a typical successful trajectory achieved in the experiments.

VI. Conclusion

A laboratory setup, consisting of an inverted pendulum to be stabilized by on/off actuators, has been designed and tested. The setup is simple, easy to build, and useful for the study of nonlinear control systems.

Centered on the pendulum stabilization, a project was proposed to a group of students of digital control, and completed with satisfactory results. The program they developed is now useful for other students, supporting a laboratory practice with nonlinear control. This practice requires little time, and is of high pedagogical impact.

For the students who participated in the project, the learning effects have been multidimensional: computer control of real plants, phase-plane and trajectories, nonlinearities, real-time control strategies, etc. New projects will be proposed, as a continuation of control ideas for the stabilization problem.

ACKNOWLEDGMENT

At least 12 students participated in the initial experiments and the project related to the new setup. The author wishes to thank all of them for their fruitful collaboration.

REFERENCES

[1] L. S. Pontryagin, V. G. Boltyanskii, R. V. Gamkrelidze, and E. F. Mishchenko, *The Mathematical Theory of Optimal Processes*. New York: Wiley, 1962.
[2] A. E. Bryson, Jr. and Y. C. Ho, *Applied Optimal Control*. New York: Wiley, 1975.
[3] F. L. Lewis, *Optimal Control*. New York: Wiley, 1986.
[4] J. C. Hsu and A. U. Meyer, *Modern Control Principles and Applications*. New York: McGraw-Hill, 1968.
[5] V. I. Utkin, "Variable structure systems with sliding modes," *IEEE Trans. Automat. Contr.*, vol. AC-22, pp. 212–222, 1977.
[6] J. J. E. Slotine and W. Li, *Applied Nonlinear Control*. Englewood Cliffs, NJ: Prentice-Hall, 1991.
[7] H. Kwakernaak and R. Sivan, *Linear Optimal Control Systems*. New York: Wiley, 1972.
[8] S. Mori, H. Nishihara, and K. Furuta, "Control of unstable mechanical system: Control of a pendulum," *Int. J. Contr.*, vol. 23, no. 5, pp. 673–692, 1976.
[9] M. Anderson and W. Grantham, "Lyapunov optimal feedback control of a nonlinear inverted pendulum," *J. Dyn. Syst. Meas. Contr.*, vol. 111, pp. 554–558, 1989.
[10] C.-S. Chen and W.-L. Chen, "Robust adaptive sliding-mode control using fuzzy modeling for an inverted-pendulum system," *IEEE Trans. Ind. Electron.*, vol. 45, pp. 297–306, 1998.
[11] K. S. Yeung and Y. P. Chen, "Sliding-mode controller design of a single link flexible manipulator under gravity," *Int. J. Contr.*, vol. 52, no. 1, pp. 101–117, 1990.
[12] H. N. Iordanou and B. W. Surgenor, "Experimental evaluation of the robustness of discrete sliding mode control versus linear quadratic control," *IEEE Trans. Contr. Syst. Technol.*, vol. 5, pp. 254–260, 1997.
[13] C. E. Lin and Y. R. Sheu, "A hybrid-control approach for pendulum-cart control," *IEEE Trans. Ind. Electron.*, vol. 39, pp. 208–214, 1992.
[14] A. G. Barto, R. S. Sutton, and C. W. Anderson, "Neuronlike adaptive elements that can solve difficult learning control problems," *IEEE Trans. Syst., Man, Cybern.*, vol. 13, pp. 834–846, Sept. 1983.
[15] C. W. Anderson, "Learning to control an inverted pendulum using neural networks," *IEEE Contr. Syst. Mag.*, vol. 9, no. 2, pp. 31–36, 1989.
[16] M. E. Magańa and F. Holzapfel, "Fuzzy logic control of an inverted pendulum with vision feedback," *IEEE Trans. Educ.*, vol. 41, pp. 165–170, May 1998.
[17] W. G. Hurley and W. H. Wölfle, "Electromagnetic design of a magnetic suspension system," *IEEE Trans. Educ.*, vol. 40, pp. 124–130, May 1997.

Jose Maria Giron-Sierra (M'94) was born in Valladolid, Spain. He received the Licentiate and Ph.D. degrees in physics from the Universidad Complutense de Madrid, Spain, in 1972 and 1978, respectively.

He is a Full Professor with the Department of Computer Architecture and Automatic Control from 1988. He has authored 60 publications in conference proceedings and journals. He holds two patents for a robot and a distributed control system. His research is related to applied automatic control and simulation: high-speed ships, airplanes and spacecrafts, cooperative robotics, logistics, process control, soft computing for optimization.

Dr. Giron-Sierra is a member of IFAC, and the Spanish Society of Pedagogy.