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A GENETIC OPTIMIZATION METHOD FOR DYNAMIC PROCESSES

B. de Andrés-Toro; J.M. Girón-Sierra; J.A. López-Orozco; J. Alvarez-Ruiz; P. Fernández Blanco

Departamento de Arquitectura de Computadores y Automática Universidad Complutense de Madrid, 28040 Madrid, Tf.: +34 91 394 43 84, Fax.: +34 91 394 46 87 Spain E-mail: deandres@eucmax.sim.ucm.es

EXTENDED ABSTRACT

The paper presents a dynamic optimization method based on Genetic Algorithms. The optimization problem considered is to find a temporal profile for the control variable, so the process, under such control, follows an optimal trajectory. The method encodes candidate solutions as chromosomes, using a piecewise linear approximation of the control profile. To show the general validity of the method, several examples are taken from optimization literature, and the method is applied, always having satisfactory results. The method has interesting operational and computational advantages.

KEYWORDS

• Optimization • Genetic algorithms • Process control • Optimal trajectory

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J. Alvarez-Ruiz, P. Fernández Blanco**

*Departamento de Arquitectura de Computadores y Automática
Universidad Complutense de Madrid, 28040 Madrid, Spain
E-mail: deandres@eucmax.sim.ucm.es, Tf.: +34 91 394 43 84, Fax.: +34 91 394 46 87*

Abstract: The paper presents a dynamic optimization method based on Genetic Algorithms. The optimization problem considered is to find a temporal profile for the control variable, so the process, under such control, follows an optimal trajectory. The method encodes candidate solutions as chromosomes, using a piecewise linear approximation of the control profile. To show the general validity of the method, several examples are taken from optimization literature, and the method is applied, always having satisfactory results. The method has interesting operational and computational advantages. *Copyright © 1999 IFAC*

Keywords: Optimization, Genetic algorithms, Process control, Optimal trajectory.

1.- INTRODUCTION

There are important industrial and laboratory processes that can be described by state variables that follow a trajectory along time. This trajectory depends on control inputs, that drive the process to obtain some desired results. Usually there are constraints for the state variables and the control inputs. In general, we have here an optimization problem, that could be multiobjective: less time, more production. The solution is in the form of a trajectory of the control inputs, that should be applied to obtain the best behaviour of the process.

If no analytical solution is available for the dynamic optimization problem at hand, there are still some algorithmic methods that could be useful. Dynamic programming is a good candidate, but has some well-known problems when the grid to be explored has many nodes (Edgar and Himmelblau, 1988). There is a line of research (Dadebo and McAuley, 1995; Bojkov and Luus, 1994) that promotes iterative dynamic programming algorithms to cope with relatively complex problems. Also, there are alternatives in the form of search methods (Gupta, 1995) that can get even better results. In general, these methods apply a discretization of the control variable(s) into a series of equal time intervals, and the optimal profile obtained consists of a series of constant horizontal steps.

Genetic Algorithms are showing interesting features to solve optimization problems, as a searching method that avoids to be trapped into local maxima. To be applied to a problem, a genetic encoding must be defined to allow for the search of solutions in terms of chromosomes.

This paper presents a method for the optimization of dynamic processes, based on genetic algorithms through the direct genetic encoding of piecewise linear aproximations of control profiles. The method is the result of some years of research related to beer fermentation (Andres-Toro et al., 1998). To show that the method has a general potential, some examples of other processes have been selected, and the method has been applied with successful results.

The first example is the beer fermentation, that constitutes a fairly complex problem. The second example has an analytical solution, as published in (Ramirez, 1994), so it is useful to show how good is

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the solution reached by our approximation. The rest of the examples are taken from the recent literature that deals with iterative dynamic programming or semiexhaustive search, and serve for comparison purposes (in every case, the method presented in this paper obtains better results).

2.- THE GENETIC-BASED METHOD

2.1.- Genetic Encoding:

Genetic Algorithms (Goldberg, 1989) are search algorithms that mimic natural selection and genetic. It differ from other searching methods in several ways: it start from a population of solutions, not a single initial solution; it work with probabilistic rules, instead of deterministic rules; it search several solutions in parallel; etc. The algorithms proceeds along several generations. A performance index is employed to select parents for each generation, and to eliminate worst members of the population.

The first step to apply Genetic Algorithms is to represent the problem in terms of chromosomes. For the kind of problems considered here, the approach taken by our method is to use a linear piecewise approximation of the profile of the control variable, as described in figure 1. The total time of the process is divided into equal time intervals. The ordered series of values of the control variable at the breakpoints, from left to right, are taken in the same order as genes, forming a chromosome

Chromosome = [119, 142, 153, 118...]

(for a profile with real values of 11.9, 14.2, 15.3...)

To ease computations, the encoding is directly made in integer numbers (not a binary representation). These numbers are obtained from real numbers by a displacement of the decimal point and rounding.

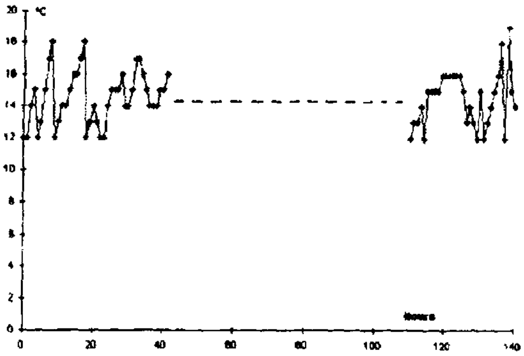

Fig. 1. a piecewise approximation of the profile of the control variable.

2.2.- Specifications of the Algorithm:

The dynamic optimization problem to be solved in each case has two main components: a model of the process (differential equations that describe its temporal behaviour, as a response to the control inputs), and a performance index (an objective function) to measure how good is any trajectory of the process.

Each chromosome represents a candidate optimal solution for the control, to drive the dynamical process along a trajectory that exhibits best characteristics. The method we propose, uses the performance index as the fitting function of the genetic algorithm. The fitting function is employed to select best chromosomes. The evaluation of each chromosome is made by running a simulation of the dynamic process: the model is used to know the response to the control variable profile described by the chromosome, and to determine the value of the performance index.

After extensive studies on what are the more convenient specifications for the Genetic Algorithm, the following values were established: an initial population of 10 individuals, mutation probability of 0.007, and crossover probability of 0.7.

The steps of the algorithm are as follows:

1. Obtain 10 individuals as initial population.
2. Evaluate the individuals according to performance index J.
3. Select the four individuals as parents, using the roulette-wheel method.
4. Obtain new offspring employing crossover and mutation operators.
5. Evaluate the new individuals.
6. Substitute four worst individuals with the new offspring.
7. If last generation, end; else, return to step 3.

2.3.- Comments and Results

The method has been implemented in Borland C++, to achieve rapid calculations. By using a modular approach to the software development, a general tool is obtained: easy to apply for different processes, changing only procedures with the model and the performance index. The method has been named Genetic Dynamic Optimization (GDO).

In all the cases we studied the results were encouraging from the very beginning. A discretization of the control variable profile as fine as desired can be applied, with a moderate computational cost. As shown in the following examples, using a Pentium II/233 the computation times are in the order of minutes, or even seconds, for the kind of discretizations employed in the literature.

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3.- APPLICATION OF THE METHOD TO EXAMPLES:

Several examples have been selected, in order to show the viability of the GDO method. The first is related with our research with the optimization of beer fermentation, and includes a relatively complex model. The second has a published analytical solution, and is a good reference to know if the result obtained by GDO is far from the true optimum. The others belong to the line of research mentioned before, that propose alternative algorithms with results that are interesting for comparison purposes.

Table 1 summarizes the main aspects concerning the GDO algorithm, calculation time, and optimization results, for each of the examples considered in this study.

Table 1. Values of the variables and results obtained for the examples.

	Ex. 1	Ex. 2	Ex 3
Genes / Chromosome	28	16	20
Population Size	10	10	10
Number Generations	1000	1000	5000
Process Duration	140h	8m	1m
Discretization (time)	5h	0.5m	0.05m
dt (integration)	0.1h	0.01m	0.01m
Computation Time	10.1m	1.38s	0.349s
Optimum Value	592.36	0.674	0.093

	Ex. 4	Ex. 5	Ex 6
Genes / Chromosome	26	20	33
Population Size	10	10	10
Number Generations	3000	2000	1000
Process Duration	26h	60m	132h
Discretization (time)	1h	0.33m	4h
dt (integration)	0.01h	0.01m	0.005h
Computation Time	14.3m	3.98m	2.76m
Optimum Value	106.34	20911.1	93.019

Example 1: The case considered is the optimization of the conventional batch fermentation of beer, keeping industrial conditions: no stirring, industrial wort and yeast. The control variable we can manipulate is temperature along time. The objective is to determine the control input to be applied, a temperature profile, such that optimizes (maximize) the yield of ethanol. There are two constraints: the concentration of ethyl acetate and diacethyl can not surpass certain limits that degrades taste and smell.

Temperature is also limited under 18ºC because of risk of spoilage by *Lactobacillus Plantarum*. The process model is explained in (Andres-Toro et al., 1997). This model distinguishes two different phases: lag phase, and fermentation phase, both with his own differential equations.

Lag phase:

$$x_{active} + x_{lag} = constant = 0.48 \cdot x_{initial} \quad (1)$$

$$dx_{active}/dt = \mu_{lag} \cdot (0.48 \cdot x_{initial} - x_{active}) \quad (2)$$

$$dx_{lag}/dt = x_{active} = -\mu_{lag} \cdot x_{lag} \quad (3)$$

Fermentation phase:

$$dx_{active}/dt = \mu_x \cdot x_{active} - k_m \cdot x_{active} + \mu_L \cdot x_{lag} \quad (4)$$

$$\mu_x = \mu_{xo} s/(0.5\, s_{initial} + e) \quad (5)$$

$$dx_{bottom}/dt = \mu_D \cdot x_+ \quad (6)$$

$$ds_{cons}/dt = \mu_s \cdot x_{active} \quad (7)$$

$$\mu_s = \mu_{so} s/(k_s + s) \quad (8)$$

$$de/dt = \mu_a \cdot f \cdot x_{active} \quad (9)$$

$$f = 1 - e/(0.5 \cdot s_{initial}) \quad (10)$$

$$\mu_a = \mu_{ao} s/(k_a + s) \quad (11)$$

$$d(ea)/dt = Y_{eas} \cdot \frac{ds}{dt} = Y_{eas} \cdot \mu_s \cdot x_{active} \quad (12)$$

$$d(vdk)/dt = k_{dc} \cdot s\, x_{active} - k_{dm} \cdot (vdk) \cdot e \quad (13)$$

As temperature is the variable we can manipulate, the main effort of our experimental research was directed to establish how it affects the model parameters. The expressions obtained are the following:

$$\mu_{xo} = 1.095 10^{47} \cdot e^{\frac{-63720}{1.99536(T+273.15)}} \quad (14)$$

$$k_m = 3.373 10^{56} \cdot e^{\frac{-76450}{1.99536(T+273.15)}} \quad (15)$$

$$Y_{eas} = 1.1129 10^{39} \cdot e^{\frac{-53056}{1.99536(T+273.15)}} \quad (16)$$

$$\mu_{D0} = 4.889 10^{14} \cdot e^{\frac{-20020}{1.99536(T+273.15)}} \quad (17)$$

$$\mu_{S0} = 6.232 10^{-19} \cdot e^{\frac{23254}{1.99536(T+273.15)}} \quad (18)$$

$$\mu_{a0} = 26.3865 e^{\frac{-2528.6}{1.99536(T+273.15)}} \quad (19)$$

$$\mu_{lag} = 2.2041 10^{13} \cdot e^{\frac{-76450}{1.99536(T+273.15)}} \quad (20)$$

$$k_a = 1.1081 10^{-52} \cdot e^{\frac{-76450}{1.99536(T+273.15)}} \quad (21)$$

Taking into account the main goals of the optimization, an objective function J, was formed adding five weighted terms:

$$J_1 = +10\, ethanol_{end} \quad (22)$$

$$J_2 = -5.73\, e^{(95\, diacethyl - 11.51)} \quad (23)$$

$$J_3 = -1.16\, e^{(460\, acetate - 66.77)} \quad (24)$$

$$J_4 = -\int_0^t 9.91 \cdot 10^{-17} \cdot e^{2.31 \cdot T} \cdot dt \quad (25)$$

$$J_5 = -\sum_{i=1}^{140} abs(T_{i+1} - T_i) \quad (26)$$

The last term, that is a simple calculation of the profile length, will smooth the temperature profile obtained by the GDO. Note also how the

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exponentials will forbid any limit violation in the final by-product concentrations. The time allowed for the fermentation is 140 hours.

The use of one objective function grouping all the aspects of the problem makes easy the application of Genetic Algorithms. It is immediate to employ it as the fitting function.

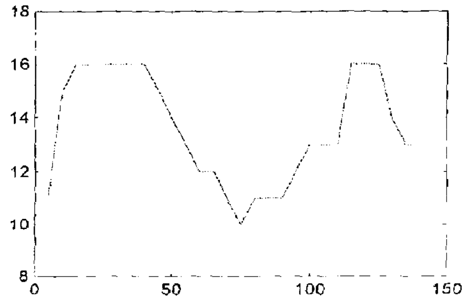

Fig.2. Optimal control policy for the example 1

The application of our method GDO renders a maximum value of 592.37. The optimal control profile obtained is represented in figure 2.

Example 2: The example is taken from the book (Ramirez, 1994), pp. 84-87. It consists of a batch reactor with two first order reactions, that was first studied by (Bilous and Amundson, 1956):

$$P \xrightarrow{k_1} Q \xrightarrow{k_2} R$$

The material balance equations for species $P(x_1)$, and $Q(x_2)$, are given by:

$$dx_1/dt = f_1 \quad (27)$$

$$dx_2/dt = f_2 \quad (28)$$

where,

$$f_1 = -k_1x_1 \quad f_2 = k_1x_1 - k_2x_2 \quad (29)$$

$$k_1 = k_{10}e^{-E_1/RT} \quad k_2 = k_{20}e^{-E_2/RT} \quad (30)$$

In (Ramirez, 1994) there is a long description of the problem, with parameter values, analytical expressions, and a figure with the optimal solutions.

The time allowed for the process is t_f = 8 min. The control variable is temperature. An optimal profile of the temperature u = T(t) must be found that maximizes the yield of species Q, x_2, at final time t_f.

Using our GDO method, a satisfactory solution was found, obtaining an optimum value of 0.6737 and spending only 1.38s of computation time. The optimum value obtained is similar to the value displayed in a figure of (Ramirez, 1994). The optimal control profile obtained with the GDO method is displayed in figure 3.

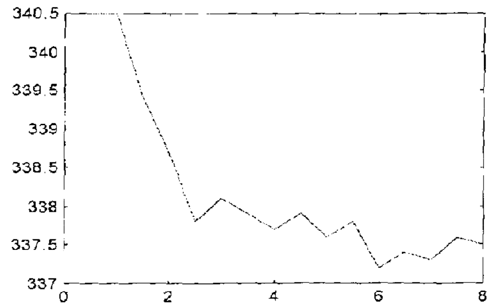

Fig.3.Optimal control policy for the example 2

Example 3: This example has been studied by (Yeo, 1980; Luus, 1990; Gupta, 1995). It is a process described by the following model:

$$dx_1/dt = x_2 \quad (31)$$

$$dx_2/dt = -x_3u + 16t - 8 \quad (32)$$

$$dx_3/dt = u \quad (33)$$

$$dx_4/dt = x_1^2 + x_2^2 + 0.0005(x_2 + 16t - 8 - 0.1x_3u^2)^2 \quad (34)$$

$$x(0) = [0 \;\; -1 \;\; -\sqrt{5} \;\; 0]^T \quad \text{and} \quad t_f = 1$$

The control is bounded by $-4 \le u \le 10$

It is required to find a control trajectory over t_f = 1 that minimizes $x_4(t_f)$.

Employing quasilinearization, (Yeo, 1980) obtained an optimum value of 0.14052. In (Luus, 1990), the same problem is attacked with IDP (iterative dynamic programming), taking 10 control steps, with 20 iterations, and computing during 11 s. on a Cray X-MP/24, to obtain an optimum value of 0.12012. And using semiexhaustive search, (Gupta, 1995) gets, with 24 iterations, very similar results: on a 486/33 computer, after 76 seconds of computation, the search reaches an optimum value of 0.12011.

In our case, employing GDO, in less than half second (0.349s) a minimum valour equal to x_4 = 0.092753 is obtained. The control profile (u) is represented in figure 4.

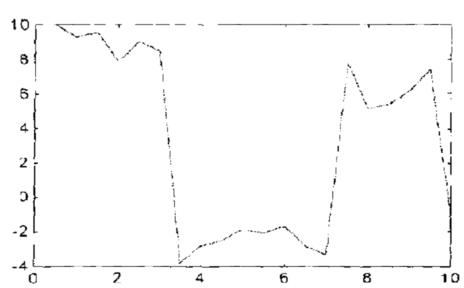

Fig.4. Optimal control for the example 3.

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Example 4: The maximization of secreted protein in a fed-batch reactor was studied first by (Park and Ramírez , 1988), and later by (Luus, 1991a).The system is described by the following equations:

$$dx_1/dt = g_1(x_2 - x_1) - (u/x_5)x_1 \quad (35)$$

$$dx_2/dt = g_2 x_3 - (u/x_5)x_2 \quad (36)$$

$$dx_3/dt = g_3 x_3 - (u/x_5)x_3 \quad (37)$$

$$dx_4/dt = -7.3 g_3 x_3 + (u/x_5)(20 - x_4) \quad (38)$$

$$dx_5/dt = u \quad (39)$$

where, $x(0) = [0\ 0\ 1\ 5\ 1]$

$$0 \le u \le 10$$

$$g_1 = 4.75 g_3/(0.12 + g_3) \quad (40)$$

$$g_2 = x_4 e^{-5x_4}/(0.1 + x_4) \quad (41)$$

$$g_3 = 21.87 x_4/((x_4 + 0.4)(x_4 + 62.5)) \quad (42)$$

The total amount of protein ($x_1(t_f)\ x_5(t_f)$) produced at $t_f = 24$ h. must be maximized. For this problem the results obtained by dynamic programming were very close to those obtained by non linear programming, as described in (Luus and Rosen, 1991).

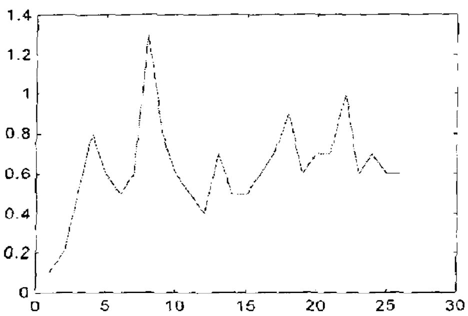

Fig.5. Optimal control policy for the example 4.

Using IDP, for a piecewise constant control trajectory over 20 time stages, and $t_f = 24$, (Luus and Rosen, 1991) reached a maximum value of 106.27 (106.28 for non linear programming), spending less than 1min of computation time on a CRAY X-MP/24.

With the GDO method, with 14.32 min. of computation time, a maximum value of 106.34 is obtained. Figure 5 displays the optimal control profile.

Example 5: Chen and Hwang, 1990; Luus, 1991b; Gupta, 1995) consider this problem. It consists of a fed-batch reactor, in which ethanol is produced by *Sacharomyces cerevisiae* and the production rate of ethanol is inhibited by itself. Assuming that at in the beginning, the fermenter is filled with 10 litres of broth and that the maximum capacity of the fermenter is 200 litres, the control variable (u) in this case is the maximum feed rate, restricted up to 10 l/hr. The total amount of ethanol ($x_3(t_f)\ x_4(t_f)$) produced at t_f=62h, must be maximized. The following differential equations describe the system:

$$dx_1/dt = g_1 x_1 - u x_1/x_4 \quad (43)$$

$$dx_2/dt = -10.g_1 x_1 + u.(150 - x_2)/x_4 \quad (44)$$

$$dx_3/dt = g_2 x_1 - u.x_3/x_4 \quad (45)$$

$$dx_4/dt = u \quad (46)$$

where: $g_1 = 0.408.x_2/((1 + x_3/16)(0.22 + x_2))$ (47)

$$g_2 = x_2/((1 + x_3/71.5)(0.44 + x_2)) \quad (48)$$

$$x(0) = [1\ 150\ 0\ 10]$$

$$0 \le u \le 10$$

$$x_4(t) \le 200$$

(Chen and Hwang, 1990) found a maximum value of 20073. For a piecewise control trajectory over 20 time stages, using IDP (Luus, 1991b) reported 20814.8. And starting from the optimal trajectory obtained by Luus, but using semiexhaustive search, (Gupta, 1995), reached a maximum value of 20830. With the GDO method, we obtain a maximum value of 20911.08, in 3.98 min of computation time. The control profile obtained is shown in figure 6.

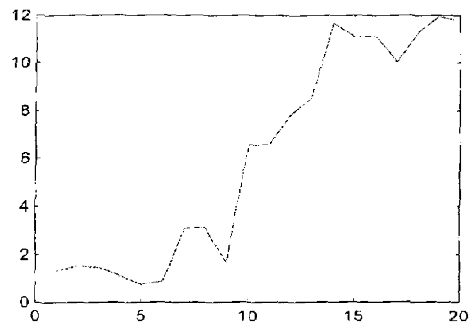

Fig.6. Optimal control policy for the example 5.

Example 6: This example consists in one fed-batch fermenter for biosynthesis of penicillin, with constraints on the state variables. Feed rate is the control variable, and we wish that the total amount of penicillin ($x_2(t_f)\ x_4(t_f)$) produced at t_f=132h is maximized. The process is described by the following equations:

$$dx_1/dt = h_1 x_1 - x_1 u/(500 x_4) \quad (49)$$

$$dx_2/dt = h_2 x_1 - 0.01 x_2 - x_2 u/(500 x_4) \quad (50)$$

$$dx_3/dt = -h_1 x_1/0.47 - h_2 x_1/1.2 - 0.029 x_1 x_3/(0.0001 + x_3) + (1 - x_3/500)u/x_4 \quad (51)$$

$$dx_4/dt = u/500 \quad (52)$$

where:

$$h_1 = 0.11 x_3/(0.006 x_1 + x_3) \quad (53)$$

$$h_2 = 0.0055 x_3/(0.0001 + x_3(1 + 10 x_3)) \quad (54)$$

$$x(0) = [1.5\ 0\ 0\ 7]^T$$

$$0 \le u \le 50$$

$$0 \le x_1 \le 40$$

$$0 \le x_3 \le 25$$

$$0 \le x_4 \le 10$$

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For this problem, (Cuthrell and Biegler, 1989) found a maximum value of 87.69. Using IDP for a trajectory over 20 time stages, (Luus, 1993) reported a maximum value of 87.948. Beginning from a control trajectory obtained by Luus, and with semihexaustive search, (Gupta, 1995) found a maximum value of 88, spending for 15 iterations, 47 min on the 486/33 PC.

In this case, the GDO method can obtain a maximum value of 93.019. Figure 7 displays the optimal control policy.

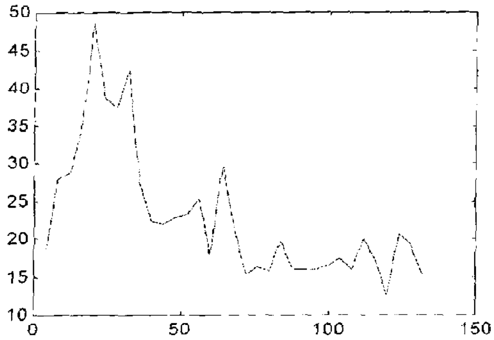

Fig.7. Optimal control policy for the example 6.

4.- CONCLUSIONS

The optimization method described in this paper provides an easy way to solve nonlinear optimal control problems. Based on Genetic Algorithms, the method is able to avoid local optimal solutions. Compared to other methods, it has advantages of simple, fast, application and moderate computational cost. By way of examples, the paper shows the potential of the method for the optimization of dynamic processes.

Because of less computation is needed, the method paves the way to study finer discretizations, in search of better approximations to the true optimal solutions. Fast versions of the method could be useful, in certain slow processes, for real-time optimization applications.

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