Cascade control is a multi-loop control scheme commonly used in chemical plants. But, as the involved processes are in general complex, with delays and non-linearities, conventional control methods are not able to guarantee the final control aims. Fuzzy Control (FLC) has been successfully applied to these applications. However, controller parameters adjustment is a critical point and in Fuzzy Control there is no systematic procedure for tuning. We propose a simple tuning strategy for these FLC based on the Relay method that allows to apply it to more complicated FLC configurations, like cascade loops.

1 Introduction

In chemical industry, the processes are usually slow, making easier their self-tuning and they do not require an accurate adjustment of the controller parameters. For these processes, the structure of control in cascade has been proved very efficient, since it allows a second loop of control that it is tuned searching for tight aims.

But the involved processes are in general complex, with delays, non-linearities; so, it is not always possible to control them with classical regulators. However, fuzzy control has been demonstrated very effective in the control of these plants.

On the other hand, the tuning of the controllers is a critical point in their performance and stability. The tuning of controllers in cascade is a tedious task, and in the case of fuzzy controllers, it is not simple since it does not exist an automatic procedure for tuning.

In this work, we propose a simple tuning method for fuzzy controllers which allows its application to more complex structures, like cascade controllers. Section 2 describes the tuning of controllers in cascade by the Relay method. In section 3, our strategy for the FLC initial adjustment is presented, based on the equivalence
between a type of FLC and the conventional ones. So, it is possible to apply the Relay method to tune FLC, as it is shown by simulation examples, and operates in a cascade scheme with fuzzy controllers. The conclusions are summarized in sect. 4.

2 Tuning of cascade controllers by the Relay method

Cascade control is a multi-loop control scheme (Figure 1). The manual tuning of controllers in cascade loops is a time-consuming task. Fuzzy logic is being use to self-tuning cascade controllers, and another systematic methods have been proposed to automatize this procedure [3]. But it also possible to apply this structure employing fuzzy controllers to cover a wider range of processes.

![Figure 1: Control in cascade](image)

In order to apply the Ziegler and Nichols tuning formulas based on the frequency response [8], there are only two parameters which are necessary: the critical gain $K_c$, and the critical period $T_c$. To determine them, a relay is introduce in the control closed-loop, to force the system oscillates in a controlled limit cycle. The period of oscillation is the critical period, and the obtained amplitude of the oscillation, $a$, makes feasible to obtain the critical gain by the expression $K_c = 4d/\pi a$, where $d$ is the amplitude of the relay [1]. For these parameters $T_c$ and $K_c$, the PID parameters are calculated to give a desired response.

Either the secondary or primary loop can be placed on the relay feedback (Figure 2). The conditions for stable limit cycles are assume for the plants. The controllers in cascade tuning is carried out first in the secondary loop, and then in the primary loop.
2.1 Self-tuning of the secondary loop

In Figure 2, the primary loop is set in the position of manual operation. The relay feedback around the secondary loop results in controlled limit cycle oscillations at the cross-over frequency of $G_2$, $w_{c2}$. The describing function $N_2$ of the second relay is given by the expression $N_2(a_2) = 4d_2/\pi a_2$, where $a_2$ is the amplitude of the oscillation at the $G_2$ output, and $d_2$ is the relay amplitude. This describing function is approximately the magnitude of $Gc_2$ at the cross-over frequency and, therefore, it is the critical gain $Kc_2$ required for tuning the controller $Gc_2$ [3]. The oscillation period is the critical period $Tc_2$. With these quantities, the parameters of $Gc_2$ are tuned using the Ziegler-Nichols formulas. Generally, a P or a PI controller is enough to control the secondary loop because it is not necessary an accurate control of this part of the system. The closed loop transfer function of the system results:

$$H_1 = \frac{G_1(s)G_2(s)Gc_2(s)Gc_1(s)}{1+Gc_1(s)G_2(s)(1+Gc_1(s)G_1(s))}$$

2.2 Self-tuning of the primary loop

Once the secondary loop is designed, the primary loop is closed with a relay around it. Controlled limit cycle oscillations are observed. The plant now operates at its phase cross-over point $w_{c1}$. The describing function of this relay $N1$ at the frequency of oscillation $w_{c1}$, can be written as the next expression, with $a_1$ the amplitude of the oscillations observed at the output of the process $G1$, and $d_1$ the relay amplitude.
\[
\frac{4d_1}{\pi \cdot a_1} = \left| \frac{1}{G_2(i\omega)G_c(i\omega)} \right|_{\omega=\omega_{cr}} \quad (2)
\]

The transfer function of the closed loop system is now:

\[
H_1 = \frac{G_1(s)G_c(s)}{1 + G_1(s)G_c(s)} \left( \frac{G_2(s)G_c(s)}{1 + G_2(s)G_c(s)} \right) \quad (3)
\]

The describing function of the relay is the critical gain required for tuning the controller \(G_c\). The critical period is the period of the limit cycle oscillation. The Ziegler-Nichols formulas can be used initially to tune \(G_c\).

3 Fuzzy controllers tuning

In Process Control area, the controller parameters adjustment is a critical point. In Fuzzy Control there is no systematic procedure for tuning; in fact, there is almost no other way but trial and error.

Under certain conditions, it has been possible to establish an equivalence between some kind of fuzzy logic controllers and non-linear conventional controllers. These results have been proved by an analytic study of the equations that describe the control action in terms of the input variables [2], [4]. The starting point is to identify the FLC input and output variables with those of a conventional controller. Figure 3 shows the basic scheme of an incremental Fuzzy-PID controller, where error \(e\), the error change \(ce\) and the second error derivative \(ac\) are the input FLC variables, and the increment of the control \(\Delta u\) is the FLC output. The parameters chosen to tune the FLC are the scale factors \(GE\), \(GR\), \(GA\) and \(GU\), gains which weight the input and output variables respectively.
The restrictions for the FLC concern the number of linguistic labels, membership functions, defuzzification method, etc. Then, the control action is linear by pieces. Therefore, some of the tuning parameters of the FLC can be reduced to the parameters of a classical PID regulator (\(K_p, T_i\) and \(T_d\)), [4], [6], and the widely studied classical tuning techniques can be applied to this type of fuzzy controllers.

For instance, the Relay Method proposed by Åström [1] can be applied to FLC to obtain its tuning parameters by means the PID controller gains (Figure 4) [5].

These FLC scale factors are calculated to get a desired behaviour. For example, if these gains are set to the values given by (4), the FLC behaviour is closed similar to a PI regulator (Figure 5). These expressions have been obtained based on Buckley’s results [7] and analysing the control law for the FLC. We have replaced the membership functions equations defined for each variable in the control output equation, given by the center of area defuzzification method. But another control
Aims can be searched, like to maintain the FLC behaviour in different zones of control, or set these gains to the stationary values [4].

\[
GR = \frac{K_p \cdot (2 \cdot L - f)}{0.5 \cdot L \cdot GU} \\
GE = \frac{K_i \cdot (2 \cdot L - f)}{0.5 \cdot L \cdot GU}
\]  

(4)

where \( f = \max(\text{GE} \cdot e(t), \text{GR} \cdot ce(t)) \) and \( L \) = center of each membership function.

We have proved this indirect tuning strategy with different model plants [6]. First, \( G_2 \) have been simulated in the secondary loop of the cascade scheme. The critical parameters are obtained by the Relay method (Figure 4). Then, a PI controller with parameters \( K_p_2 = 0.3 \) and \( K_i_2 = 1 \) or the equivalent FLC is autotuned for the secondary loop using Ziegler-Nichols rules. With the secondary controller in place, the primary loop \( G_1 \) is placed on relay feedback, and the computed gains are \( K_p_1 = 0.12 \) and \( T_i_1 = 1.2 \).
This adjustment may be improved to include other heuristic aspects about the expert knowledge to emphasise the FLC non-linear characteristic.

4 Conclusions

Fuzzy Controllers have a non-linear behaviour that makes them an useful tool for the chemistry industry. But in Process Control Area, stability, efficiency, performance, etc., depend on how the controller parameters are selected. In Fuzzy Control there is no systematic procedure for tuning.

We have proposed a simple tuning strategy for FLC. Simulating the FLC behaviour, and under certain conditions, it has been possible to establish an equivalence between the FLC initial parameters and the classical PID tuning gains (as we have proved also analytically). This result allows to apply well-know tuning strategies, as the Relay method, to more complicated configurations of FLC, like cascade loops. Moreover, the plant operator feels comfortable with these new controllers because he works with PID-look-and-feel ones.

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References